MICRO-428: METROLOGY

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MICRO-428: METROLOGY

WEEK TWO: OPTICAL IMAGE SENSORS

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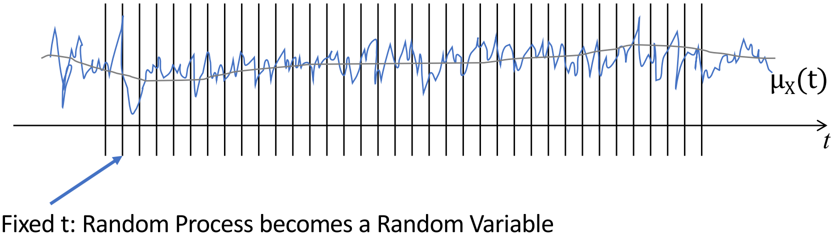
Reference Books

- A. Theuwissen, “Solid-State Imaging with CCDs”, SSS&T Library, 1995
- P. R. Gray, P. J. Hurst, S. H. Lewis, R. G. Meyer, “Analysis and Design of Analog Integrated Circuits (4th ed.)”, Wiley, 2001
- P. E. Allen, D. R. Holberg, “CMOS Analog Circuit Design (2nd ed.)”, Oxford University Press, 2002

Outline

2.0.1 Random Process – Example

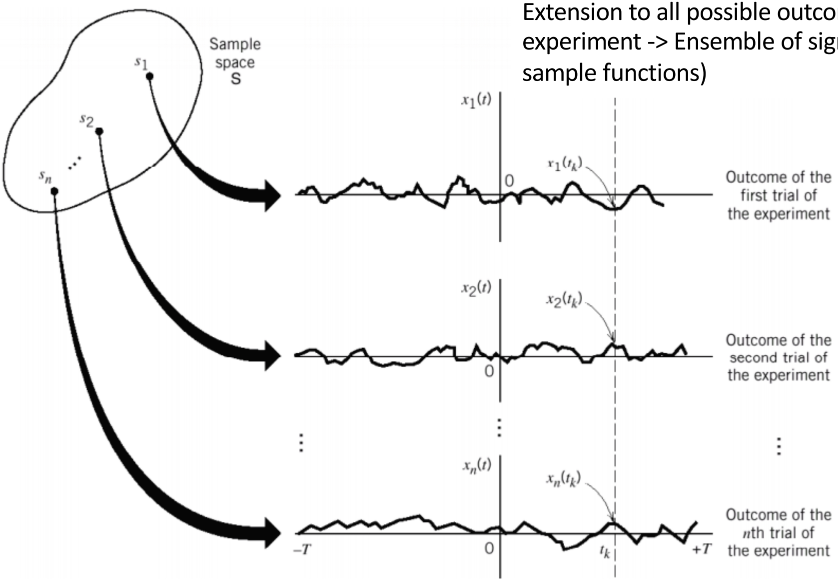
- Example (from Week 3): **Noise** is generally modeled as a random process, i.e. a collection of random variables, one for each time instant *t* in interval
- How are the RV between one another?
- How is $\mu_X(t)$ in a WSS process? And the autocorrelation function?



- How to estimate $\mu_X(t)$ and the autocorrelation function in a WSS process?

2.0.1 Random Process – Example

Extension to all possible outcomes of the underlying random experiment -> Ensemble of signals (= set of all possible sample functions)



2.0.1 Random Process (contd.) – Characterization/1

- A Random Process is characterized by the same functions already explained for RVs, but which now depend on ***t***, i.e.:

CDF: $$F_X(x,t) = P\{X(t) \leq x\}$$

X(t)= random variable at time *t*

PDF: $$f_X(x,t) = \frac{dF_X(x,t)}{dx}$$

Mean: $$m_X(t) = \overline{X(t)} = E\{X(t)\} = \int_{-\infty}^{\infty} x\, f_X(x,t)\, dx$$

Second Order Moment: $$\overline{X^2(t)} = E\{X^2(t)\} = \int_{-\infty}^{\infty} x^2\, f_X(x,t)\, dx$$

Variance: $$Var\{X(t)\} = E\left\{\left(X(t) - m_X(t)\right)^2\right\} = \int_{-\infty}^{\infty} \left(x - m_X(t)\right)^2 f_X(x,t)\, dx$$

Ensemble averages

2.0.1 Random Process (contd.) – Characterization/2

- However, in order to characterize a RP, we need to introduce two more functions, e.g. to indicate how rapidly a RP changes in time:

$$\text{Auto}-\text{covariance:} \qquad C_{XX}(t_1,t_2) = Cov\{X(t_1), X(t_2)\}$$

$$\text{Auto}-\text{correlation:} \qquad K_{XX}(t_1,t_2) = E\{X(t_1)\cdot X(t_2)\}$$

$$\text{NB:} \qquad C_{XX}(t_1,t_2) = E\{[X(t_1)-m_X(t_1)][X(t_2)-m_X(t_2)]\} = \\ = K_{XX}(t_1,t_2) - m_X(t_1)m_X(t_2)$$

NB: in general, the *autocorrelation* is the correlation of the signal with a delayed copy of itself (similarity between observations as a function of the time lag between them) [Wikipedia "autocorrelation"]

- In a similar way we can also define:

$$\text{Cross}-\text{covariance:} \qquad C_{XY}(t_1,t_2) = Cov\{X(t_1), Y(t_2)\}$$

$$\text{Cross}-\text{correlation:} \qquad K_{XY}(t_1,t_2) = E\{X(t_1)\cdot Y(t_2)\}$$

Cross-correlation: same but between two series (here: sample functions)

2.0.2 Stationary Random Process

- We characterize the RP on how their statistical properties change in time. If they do not change, we call the RP stationary. Hence:

$$f_X(x,t) = f_X(x)$$

$$m_X(t) = \overline{X(t)} = E\{X(t)\} = \int_{-\infty}^{\infty} x\, f_X(x,t)\, dx = \mu_X$$

$$Var\{X(t)\} = E\left\{\left(X(t) - m_X(t)\right)^2\right\} = \int_{-\infty}^{\infty} \left(x - m_X(t)\right)^2 f_X(x,t)\, dx = \sigma^2$$

- Weaker form: in Wide-Sense Stationary RPs, in addition to a constant mean, the autocorrelation function only depends on the time difference, but not on the absolute position in time:

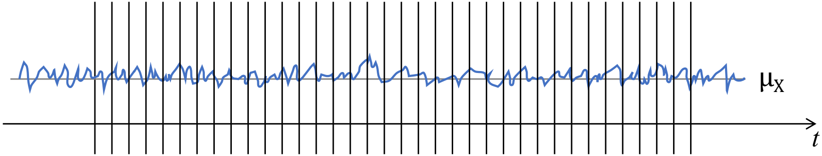
$$K_{XX}(t, t+\tau) = K_{XX}(\tau)$$

2.0.2 Stationary Random Process – Example

$$f_X(x,t) = f_X(x)$$

$$m_X(t) = \overline{X(t)} = \mu_X$$

$$Var\{X(t)\} = \sigma^2$$



2.0.2 Stationary Random Process (contd.)

- For a Wide-sense Stationary Random Process $X(t)$, the autocorrelation function has the following properties:

1. $K_{XX}(0) = E\{X^2(t)\} = \overline{X^2(t)} \geq 0$ (example: $K_{XX}(0)$ = *total power* of random signal $X(t)$)

2. $K_{XX}(\tau) = K_{XX}(-\tau)$

3. $\lim_{|\tau|\to\infty} K_{XX}(\tau) = \lim_{|\tau|\to\infty} E\{X(t) \cdot X(t+\tau)\} =$

   $= E\{X(t)\}\, E\{X(t+\tau)\} = \overline{X(t)}^{\,2}$ (example: *average or DC power* of random signal $X(t)$)

4. $|K_{XX}(\tau)| \leq |K_{XX}(0)| \qquad \text{for all } \tau$

2.1.1 Introduction to Optical Image Sensors

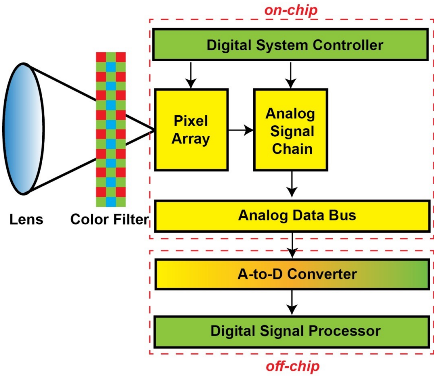
2.1.1 Introduction to Optical Image Sensors – History

1947	First transistor invented	Bell Labs
1963	First MOS image senor invented	Morrison
1964	First scanistor for image sensor	Horton (IBM)
1966	50X50 phototransistor array	Westinghouse
1967	Photon flux integration mode first proposed	Fairchild
1968	100X100 photodiode array	Fairchild
1971	First charge coupled device invented	Bell Labs
1991	First 4 MOSFET active pixel invented	Techinion
1993	128X128 active pixel array	JPL
1996	1M active pixel array	AT&T, JPL, National
1998	First single-chip camera	Lucent, VVL
2002	14 Mpixel CMOS for DSC	Fillfactory
2007	54 Mpixel CMOS for DSC	Canon
2008	120 Mpixel CMOS for DSC	Canon
2009	0.9um pixel size	Sony
2009	First consumer BSI sensor	Sony

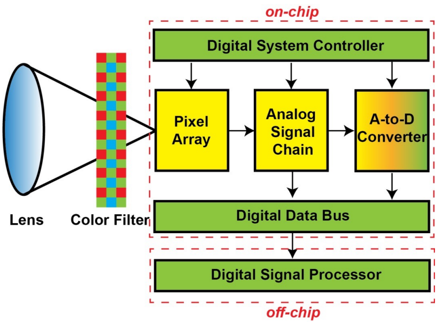
2.1.1 Introduction to Optical Image Sensors – History

2010	First commercial 3D vision camera	Canesta/Microsoft
2015	First proximity sensor based on SPAD technology	STMicroelectronics
2019-20	First commercial 3D vision camera based on SPAD	Sony/Apple
…		

2.1.1 Introduction to Optical Image Sensors – Architecture

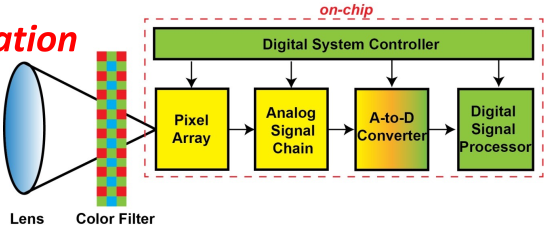


1st generation



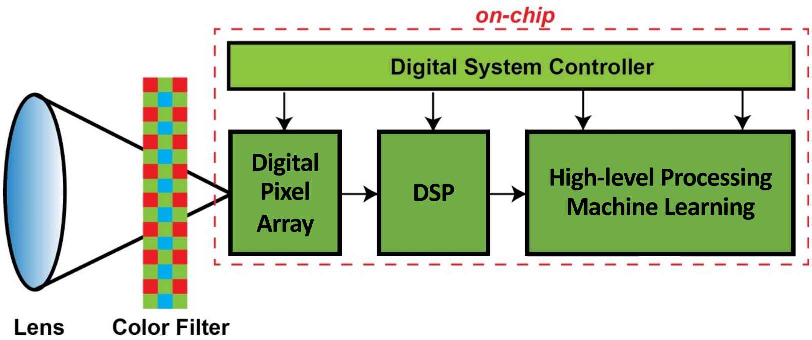
2nd generation

3rd generation



2.1.1 Introduction to Optical Image Sensors – Architecture

4th generation



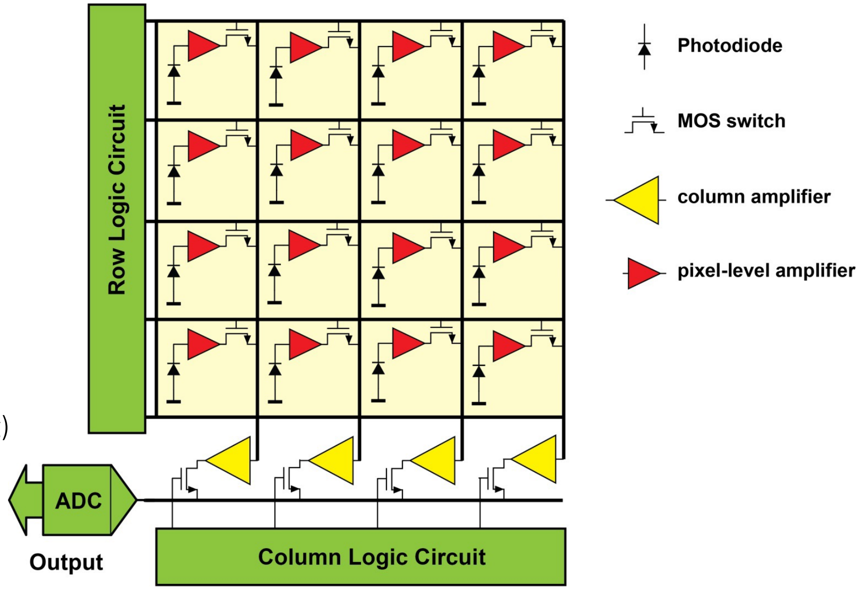
2.1.1 Introduction to Optical Image Sensors – Specifications

Specification	Example	Unit	Comments
Resolution	1024 x 1024	pixels	
Pixel noise	100	µV	
Noise readout path	150	µV	
Saturation level	20,000	electrons	
Input referred noise	183.3	µV	2.88 DN* 3.61 electrons
Dynamic range	74.9	dB	
Conversion gain	500	µV/e-	
Dark current		nA	
PRNU		%	Photo response nonuniformity
Power consumption		W	

*) DN = digital number, the number the camera assigns to a certain number of electrons detected by the camera.
In this case, 2.88 DN corresponds to 3.61 electrons => 1DN = 0.80 electrons

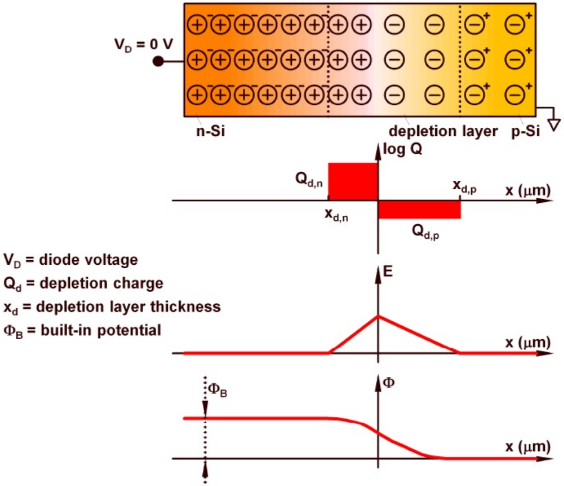
2.1.2 Introduction to Optical Image Sensors – Active Pixel Sensor (APS)

- (Digital) active pixel sensor
  - Reverse-biased photodiode
  - Switch to connect to column
  - Sequential analog readout
- Digital output
- Pros
  - Full integration (digital out)
  - No resistive losses
- Cons
  - Charge losses (charge sharing)
  - Medium speed



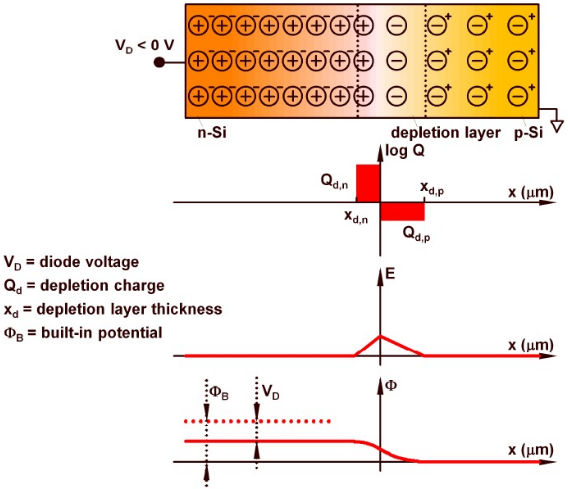
2.1.3 Introduction to Optical Image Sensors – Photodiode

- Junction in equilibrium



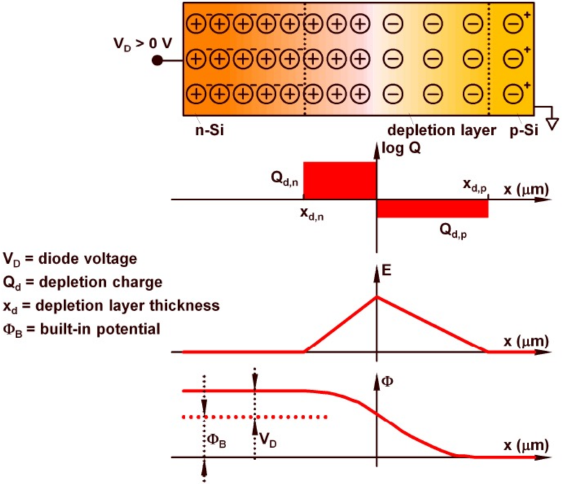
2.1.3 Introduction to Optical Image Sensors – Photodiode

- Junction forward-biased



2.1.3 Introduction to Optical Image Sensors – Photodiode

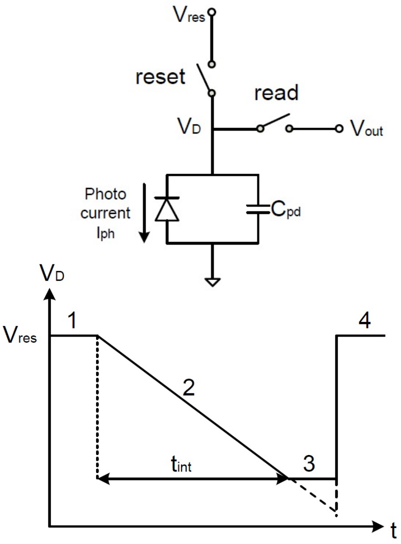
- Junction reverse-biased



2.1.3 Introduction to Optical Image Sensors – Photodiode

- Charge integration m ode

1. “Reset” close, $V_D = V_{pix}$
2. “Reset” open, V_D begins to drop
3. “Read” close, $V_{out} = V_D$
4. “Read” open, “reset” close, repeat for next frame



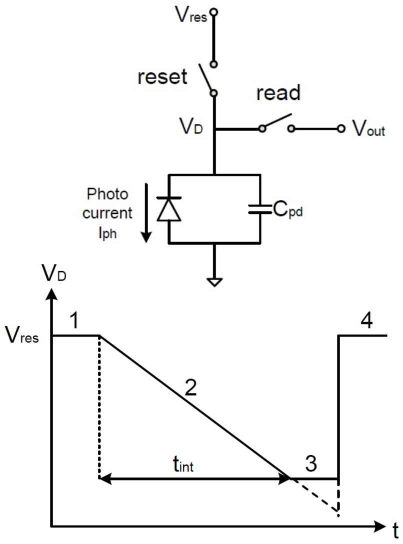
2.1.3 Introduction to Optical Image Sensors – Photodiode

- Charge integration mode

$$C_{pd}(V)\frac{dV_D(t)}{dt}=-I_{ph}$$

$$C_{pd}(V)=\frac{A_{pd}}{2}\left[\frac{2q\varepsilon_{Si}N_A}{V_D(t)}\right]^{\frac{1}{2}}$$

V_{res} : reset voltage
V_D : diode voltage
C_{pd} : diode capacitance
I_{ph} : photo current
A_{pd} : diode area
q : elementary charge
ε_{Si} : silicon permittivity
N_A : doping concentration



2.1.3 Introduction to Optical Image Sensors – Photodiode

$$\frac{A_{pd}}{2}(2q\varepsilon_{Si}N_A)^{\frac{1}{2}}\times 2\left|\sqrt{V_D}\right|_{V_{res}+\Phi_B}^{V_D(t)+\Phi_B}=-I_{ph}t$$

$$V_D(t)=\left[V_{res}^{\frac{1}{2}}-\frac{I_{ph}t}{A_{pd}(2q\varepsilon_{Si}N_A)^{\frac{1}{2}}}\right]^2$$

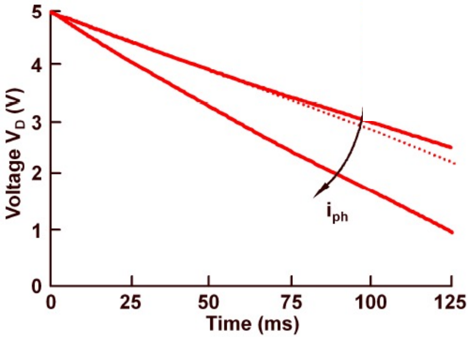
With:

$$I_{ph}=I_0A_{pd}$$

$$V_D(t)=\left[V_{res}^{\frac{1}{2}}-\frac{I_0t}{(2q\varepsilon_{Si}N_A)^{\frac{1}{2}}}\right]^2$$

Φ_B : diode built-in voltage

I_0 : incident photon flux



I_{ph} = 1pA

A_{pd} = $10\times10\mu m^2$

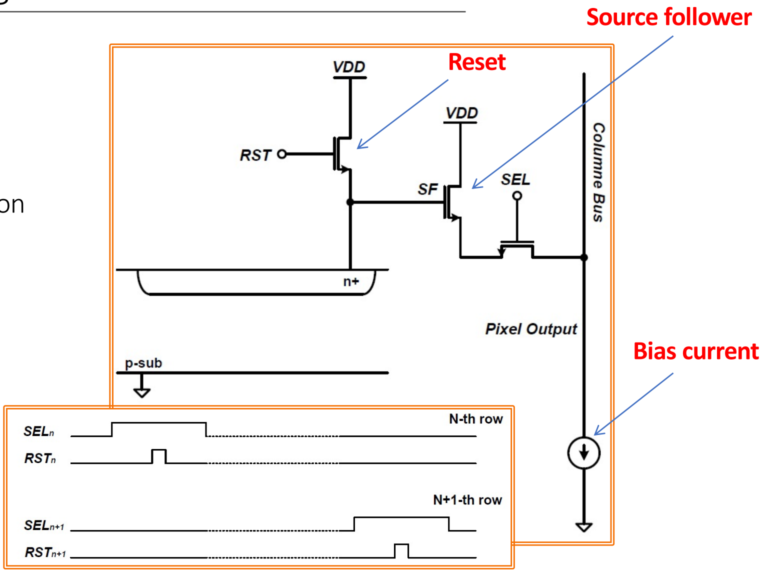
N_A = $10^{16}/cm^3$

V_{res} = 5V

2.1.4 Introduction to Optical Image Sensors – APS Pixel

- Active Pixel Sensor (APS) pixel:
  - 3 transistor per pixel
  - Reset
  - Source follower (SF) – current amplification
  - Column-level bias current

- Main limitations:
  - Noise (time-varying & time-invariant)
  - Charge sharing

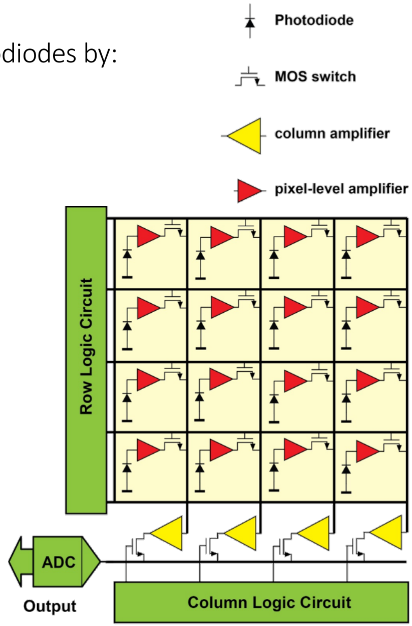


Full-well capacity defines the maximum amount of collected charges in each pixel

2.2.1 Time-varying vs. Time-invariant Noise

- Time-varying noise – time-domain random process, dominated in photodiodes by:
  - Shot noise
  - kT/C noise
  - Thermal noise (pixel-level amplifier – source follower)
  - 1/f noise

- Time-invariant noise – space-domain random process, dominated by:
  - Pixel property mismatches
  - Reset charge injection
  - Follower mismatches
  - Current bias mismatches

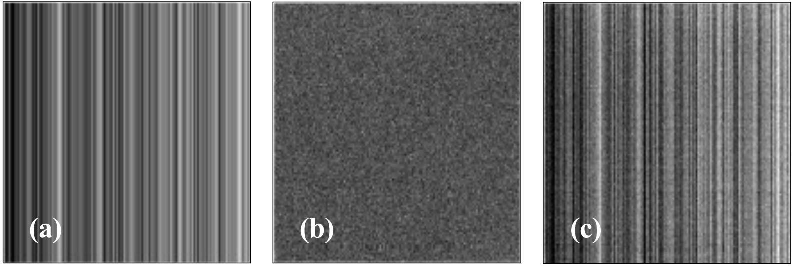


2.2.1 Time-varying vs. Time-invariant Noise

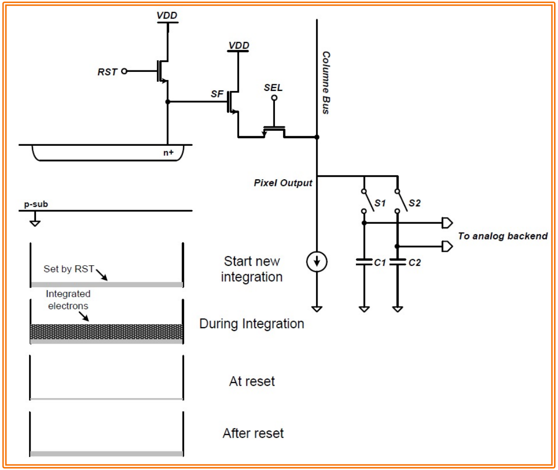
- Time-varying noise creates the visual equivalent of white noise
- Shot and thermal noise cannot be eliminated
- kT/C can be suppressed by correlated double-sampling
- 1/f noise can be *partially* suppressed by (un)correlated double sampling

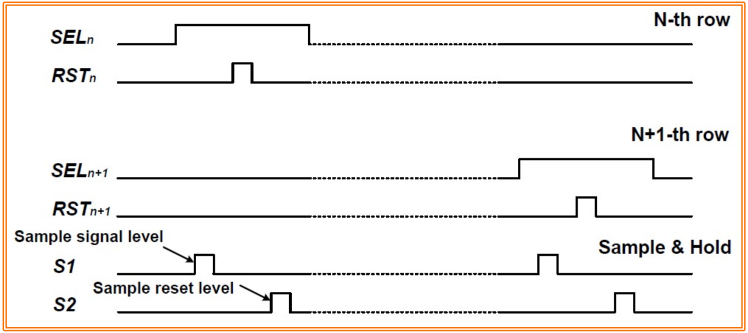
- Time-invariant noise is known (collectively) as fixed-pattern noise:
  - (a) Column-based
  - (b) Pixel-based
  - (c) Pixel/column combined

- *In principle*, it can be eliminated



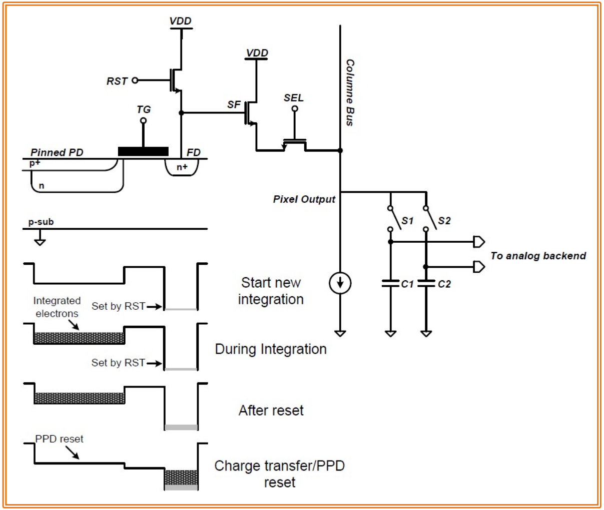
2.2.2 Noise Suppression – Uncorrelated Double-sampling

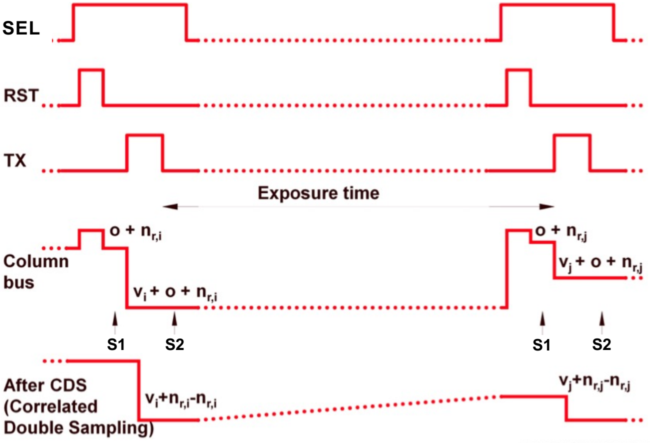




- Uncorrelated double-sampling concept
- Enables suppression of charge injection ***but not*** kT/C
- Enables partial suppression of 1/f noise

2.2.2 Noise Suppression – Pinned PD & Correlated Double Sampling

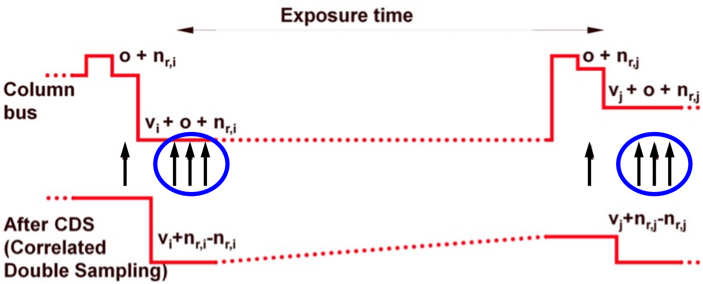




- Correlated double sampling concept
- Enables suppression of charge injection & kT/C
- Enables partial suppression of 1/f noise

2.2.2 Noise Suppression – Non-destructive Multiple Sampling

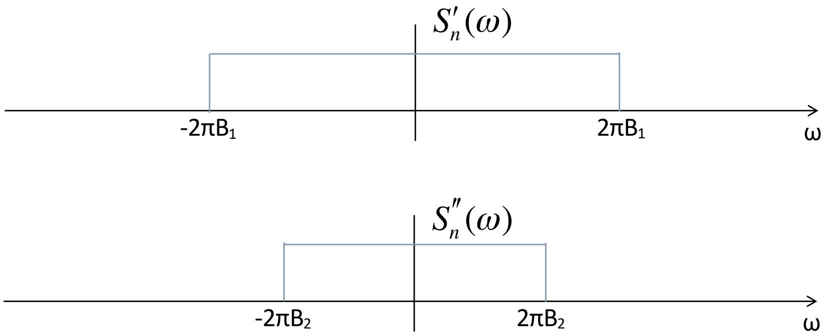
- Multiple sampling (with averaging) is equivalent to discrete integration or low-pass filtering



- Non-destructive MS + CDS is effective in reducing:
  - Uncorrelated (white) noise
  - Correlated (kTC) noise & 1/f noise

2.2.2 Noise Suppression – Low-pass Filtering

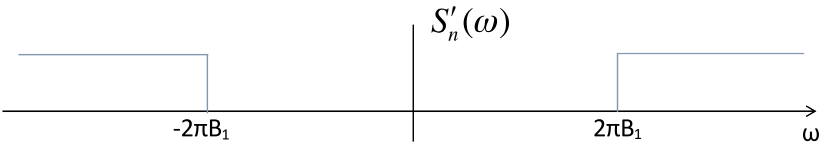
- Low-pass filtering limits the band of noise from above; effective with white noise!



$$\sigma_n^2 = 4kTR \cdot B_2 < 4kTR \cdot B_1 \quad \text{where} \quad B_1 > B_2$$

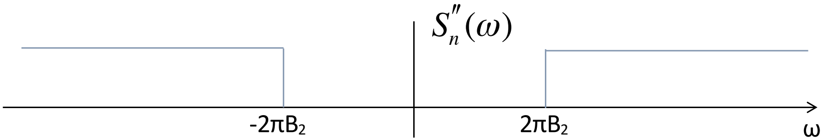
2.2.2 Noise Suppression – High-pass Filtering

- High-pass filtering limits the band of noise from below; effective with white noise!



Removed resistor-generated thermal noise:

$$\sigma_n^2 = 4kTR \cdot B_1$$



Removed resistor-generated thermal noise:

$$\sigma_n^2 = 4kTR \cdot B_2$$

where $B_1 > B_2$

2.2.2 Noise Suppression – CDS vs. High-Pass Filtering (HPF)

- Assuming CDS is performed at a time spacing T, then

$$CDS(t) = \delta(t) - \delta(t+T)$$

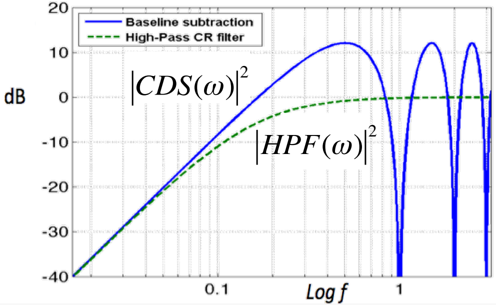
$$CDS(\omega) = 1 - e^{j\omega T} = 1 - \cos\omega T - j\sin\omega T$$

$$\left|CDS(\omega)\right|^2 = \left[1 - \cos\omega T\right]^2 + \sin^2\omega T = 2\left[1 - \cos\omega T\right]$$

- If the video signal on the diode is slow w.r.t. T, i.e. $\omega T \ll 1$, then:

$$\cos\omega T \approx 1 - \frac{1}{2}\omega^2 T^2 \rightarrow \left|CDS(\omega)\right|^2 \cong \omega^2 T^2$$

- Thus, CDS is similar to HPF:



BODE DIAGRAM highlights the low-freq cutoff

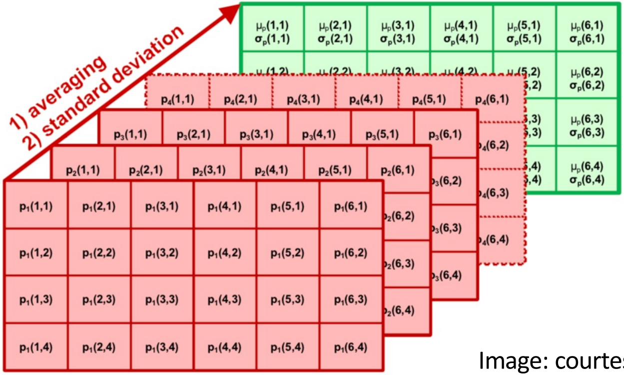
Examples with equal cutoff T=RC plotted for T=1

2.2.2 Noise Suppression – Time-varying Summary

	Filtering (high-pass, band-pass)	CDS
Uncorrelated (white) noise	✓	✗
Correlated (kT/C) noise	✗	✓
1/f noise (very slow, approaching correlated noise)	✗	✗ (✓)

2.2.3 Noise Suppression – Fixed-pattern Noise Calibration

- Compute average and standard deviation for each pixel over N frames
- The FPN value, $p_{FPN}(x, y)$, corresponds to $\mu_p(x, y)$ for pixel $p(x, y)$.
- The standard deviation, $\sigma_p(x,y)$, corresponds to the variance of the pixel, which characterizes its time-varying performance (and it is generally dominated by shot noise)



Average and standard deviation:

$$p_{FPN}(x,y) = \mu_p(x,y) \approx \frac{1}{N}\sum_{n=1}^{N} p_n(x,y) \qquad (1)$$

$$\sigma_p^2(x,y) \approx \frac{1}{N}\sum_{n=1}^{N}[p_n(x,y) - \mu_p(x,y)]^2 \qquad (2)$$

Image: courtesy of Albert Theuwissen

2.3.1 How to Measure an Optical Image Sensor

- One should always calibrate the sensor first and subtract $p_{FPN}(x,y)$ from $p_n(x, y)$ in the dark, the resulting pixel level is $p^*_n(x, y) = p_n(x, y) - p_{FPN}(x,y)$.
- Next, one can compute the dark noise as

$$\mu_{DARK} \approx \frac{1}{XYN} \sum_{y=1}^{Y} \sum_{x=1}^{X} \sum_{n=1}^{N} p_n^*(x,y) \qquad (3)$$

$$\sigma^2_{DARK}(x,y) \approx \frac{1}{XYN} \sum_{y=1}^{Y} \sum_{x=1}^{X} \sum_{n=1}^{N} [p_n^*(x,y) - \mu_p(x,y)]^2 \qquad (4)$$

- In principle, $\mu_{DARK} = 0$, while σ_{DARK} indicates the overall dark noise in binary digits if the sensor has a digital output, otherwise µV. From the binary digits the voltage can be found knowing the converter resolution and/or the overall range
- For example, a resolution (LSB) of 10µV yields 150µV for a binary digit count of 15.

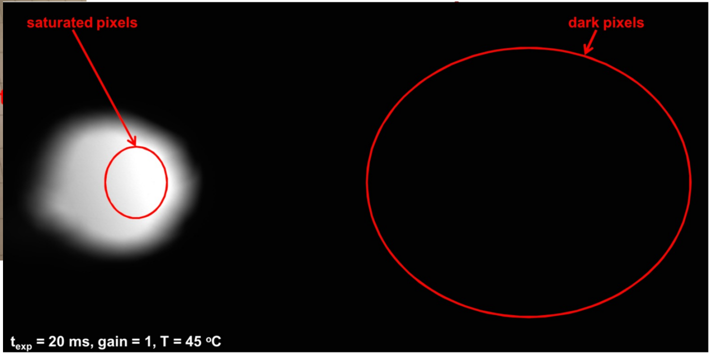
2.3.1 How to Measure an Optical Image Sensor

Specification	Example	Unit	Comments
Resolution	1024 x 1024	pixels	
Pixel noise	100	µV	
Noise readout path	150	µV	
Saturation level	20,000	e-	
Input referred noise	183.3	µV	2.88 DN (digital number) 3.61 electrons
Dynamic range	74.9	dB	
Conversion gain	500	µV/e-	
Dark current	100	pA	
PRNU	0.1	%	Photo response nonuniformity
Power consumption	200	mW	At a given speed
Frame rate	30	fps	Frames-per-second

2.3.1 How to Measure an Optical Image Sensor

- To capture the saturation level one must capture a scene with high light and dark points. As an example, consider the figures below.

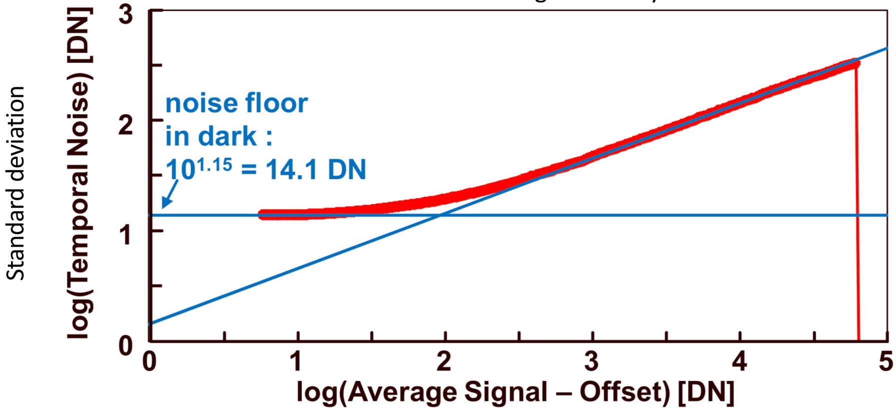
Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

- Using (2) from *N* output frames of the camera, one should compute $\sigma_p(x, y)$ for all (x, y) and plot them as a function of $\mu_p(x, y)$. For coherency one *can use* digital number (DN) – in case of a digital camera – otherwise A/D conversion should be performed on the oscilloscope and computed off-line
- Plot the curve as shown, extract noise floor

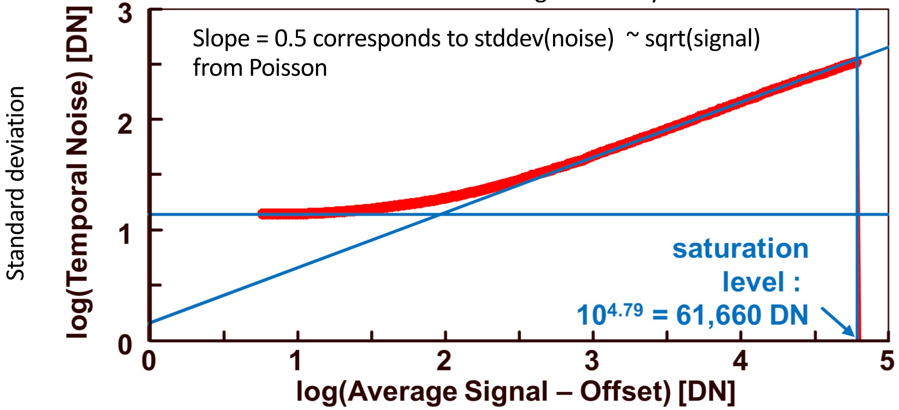
Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

- Using (2) from *N* output frames of the camera, one should order the outputs per pixel in increasing digital number (DN) – in case of a digital camera – otherwise an A/D conversion should be performed on the oscilloscope and (2) computed off-line
- From the plot, extract saturation level

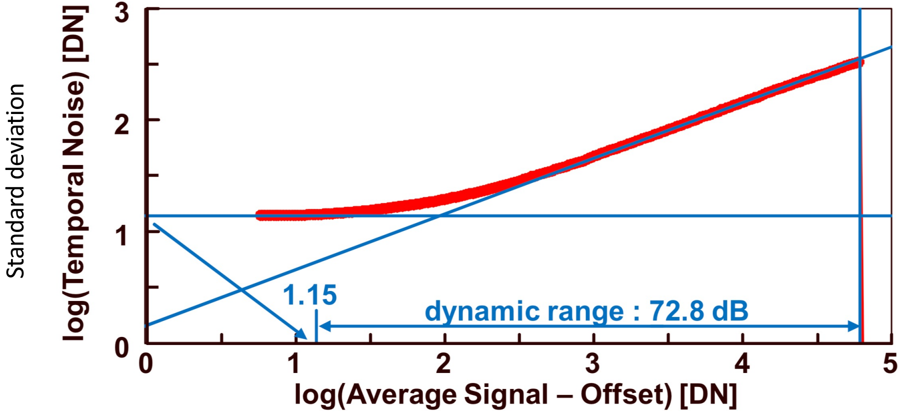
Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

- Using (2) from *N* output frames of the camera, one should order the outputs per pixel in increasing digital number (DN) – in case of a digital camera – otherwise an A/D conversion should be performed on the oscilloscope and (2) computed off-line
- From the plot, extract dynamic range

Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

- **The output signal of the sensor/camera can be written as :**

$$S_{tot} = S_o + S_d + S_{off} = k \cdot N_o \cdot t_{exp} + k \cdot N_d \cdot t_{exp} + S_{off}$$

  - S_{tot} : output signal of the sensor [DN],
  - S_o : light signal at the output [DN],
  - S_d : dark signal at the output [DN],
  - S_{off} : offset of the output signal [DN],
  - k : conversion gain [DN/e$^-$],
  - N_o : light signal [e$^-$/(pixel·s)],
  - N_d : dark current [e$^-$/(pixel·s)],
  - t_{exp} : exposure time [s].

- **The temporal noise ~~(in dark)~~ measured at the output is :**

$$\sigma_{temp}^2 = \sigma_o^2 + \sigma_d^2 + \sigma_e^2$$

  - σ_{temp} : temporal noise at the output [DN],
  - σ_o : photon shot noise [DN],
  - σ_d : dark current shot noise [DN],
  - σ_e : noise of the electronic parts [DN].

source: Albert Theuwissen

2.3.1 How to Measure an Optical Image Sensor

- **The photon shot noise can be written as :**

$$\sigma_o = k \cdot (N_o \cdot t_{exp})^{0.5}$$

- **The dark current shot noise can be written as :**

$$\sigma_d = k \cdot (N_d \cdot t_{exp})^{0.5}$$

- **Combining the formulas gives :**

$$\sigma_{temp}^2 = k^2 \cdot N_o \cdot t_{exp} + k^2 \cdot N_d \cdot t_{exp} + \sigma_e^2$$

$$= k \cdot (k \cdot N_o \cdot t_{exp} + k \cdot N_d \cdot t_{exp}) + \sigma_e^2$$

$$= k \cdot (S_{tot} - S_{off}) + \sigma_e^2$$

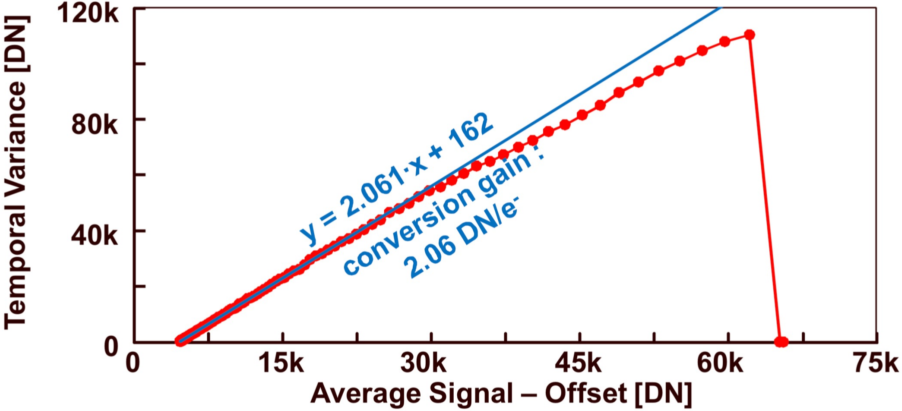
So, if the measured temporal variance is plotted as a function of the measured output signal (corrected for the offset), the conversion gain can be found as the slope of the "Mean-Variance Curve".

source: Albert Theuwissen

2.3.1 How to Measure an Optical Image Sensor

- Using (2) from *N* output frames of the camera, one should order the outputs per pixel in increasing digital number (DN) – in case of a digital camera – otherwise an A/D conversion should be performed on the oscilloscope and (2) computed off-line
- From the plot, extract conversion gain

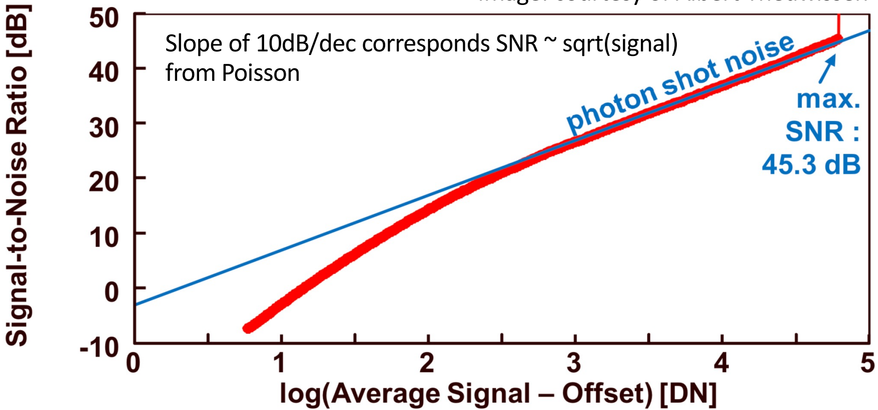
Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

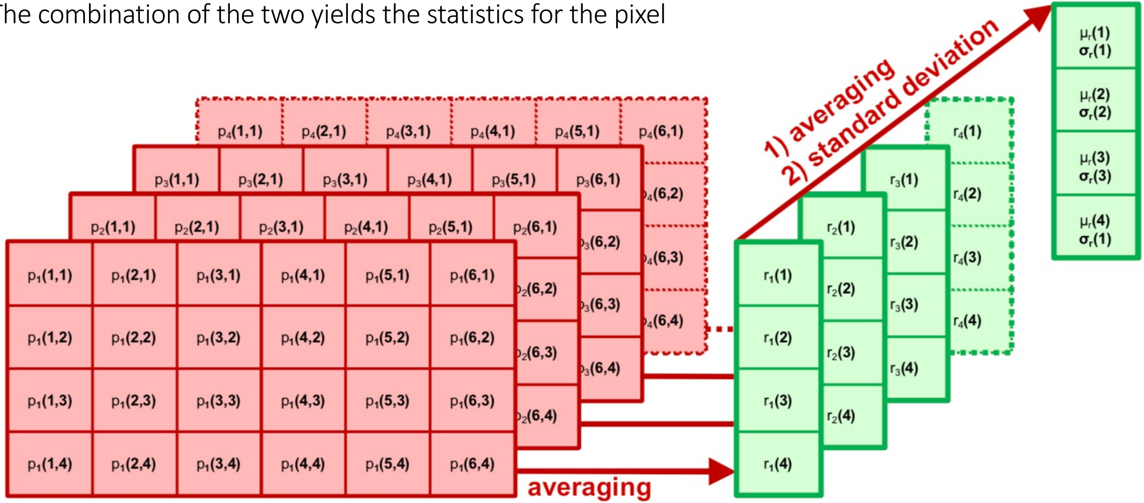
- Using (2) from *N* output frames of the camera, one should order the outputs per pixel in increasing digital number (DN) – in case of a digital camera – otherwise an A/D conversion should be performed on the oscilloscope and (2) computed off-line
- From the plot, extract SNR

Image: courtesy of Albert Theuwissen



2.3.1 How to Measure an Optical Image Sensor

- Using averaging and standard deviation calculations, extract row and column uniformity
- The combination of the two yields the statistics for the pixel



2.3.2 How to Measure an Optical Image Sensor - Summary

Specification	Meas. Value	Unit	Comments
Noise floor	4.8	e-	
Saturation level	32,200	e-	
Dynamic range	76.5	dB	
Conversion gain	2.06	DN/e-	
Max. SNR	45.3	dB	
Row FPN	0.21	e-	
Column FPN	0.29	e-	
Pixel FPN	3.08	e-	
Row temporal noise	1.17	e-	
Column temporal noise	0.18	e-	
Pixel temporal noise	6.23	e-	

Data: courtesy of Albert Theuwissen

2.4.1 Dynamic Range – Definition

- We define dynamic range as the ratio between the brightest and the dimmest object we can detect
- The generally accepted mathematical definition is:

$$DR = 20\log\left[\frac{N_{SAT}}{N_n}\right],$$

where

N_{SAT}: max. number of photocarriers at saturation.

N_{n}: min. number of photocarriers that can be detected.

2.4.1 Dynamic Range – Definition

- Due to the existence of Poisson noise in the signal, we should, more correctly, include it in the definition:

$$DR = 10\log\left[\frac{N_{SAT}}{\sqrt{N_n^2 + N_{SAT}}}\right].$$

- In addition, we should use multiplication factor 10, given the direct proportionality of N’s to photon fluxes, and thus to power.

<u>*Caveat:*</u> in the imaging community though, the Poisson noise term is generally omitted and the multiplication factor is 20 (not 10), for historical reasons.

2.4.1 Dynamic Range – In Practice

- In nature the dynamic range in the visible range is above 100dB
- The human eye can have a dynamic range up to 90dB
- CMOS APS imagers generally have a dynamic range of 40-70dB (single exposure) but they can extend it using a number of techniques

2.4.1 Dynamic Range – Example

- Same scene taken with different DR and integration time

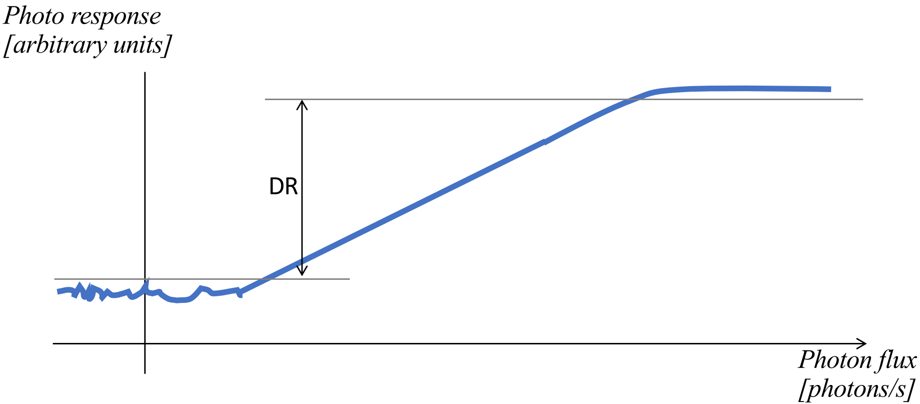
a) 60dB DR and a short integration time ("Ibis 4 imager")

b) 60dB DR and a long integration time ("Ibis 4 imager")

c) 120dB DR ("Fuga imager")

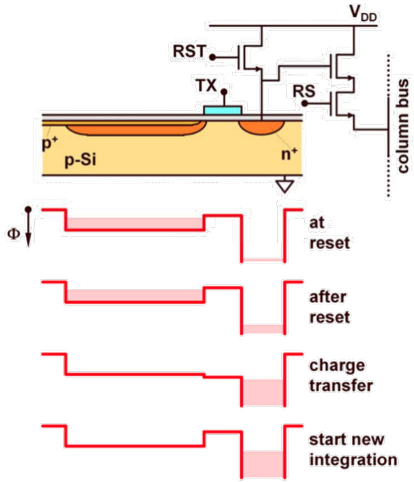
2.4.1 Dynamic Range – Limitations

- In solid-state detectors the DR is limited from above by the well capacity
- From below by the sum of all noise sources.



2.4.1 Dynamic Range – Well Capacity

- The well capacity is the largest number of charges that can be stored in the pixel
- The dominant storage unit is usually the floating diffusion in CMOS APS and CCDs
- In photon counters it is the counting device or the readout speed
- How to compute the well capacity in a photodiode?



2.4.1 Dynamic Range – Well Capacity

- Accumulation:

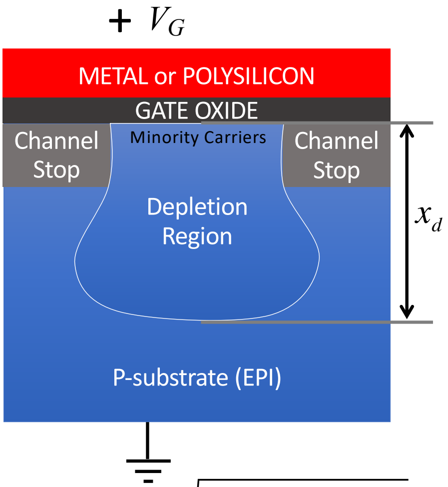
$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

- Depletion capacitance:

$$C_{dep} \cong \frac{\varepsilon_{si}}{x_d}$$

- Gate capacitance

$$C_T = \left(\frac{1}{C_{ox}} + \frac{1}{C_{dep}} \right)^{-1}$$



$$x_d = -\frac{\varepsilon_{si}}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_{si}}{C_{ox}} \right)^2 + \frac{\varepsilon_{si} V_G}{2qN_A}}$$

2.4.1 Dynamic Range – Well Capacity

- Definition: the charge Q required to bring the surface potential to zero

$$\Delta V = -\frac{Q}{C_{ox} + C_{dep}}$$

$$\text{Generally: } C_{ox} > C_{dep} \Rightarrow Q \cong C_{ox} V_{surface}$$

$$V_{surface} = \frac{qN_A}{2\varepsilon_{si}} x_d^2 \rightarrow Q \cong C_{ox} \frac{qN_A}{2\varepsilon_{si}} x_d^2$$

- Example: $N_{\mathrm{A}} = 10^{15}\mathrm{cm}^{-3}$, $V_{\mathrm{G}} = 10\mathrm{V}$, Area = $4\mathrm{x}8\mu\mathrm{m}^2$, $C_{\mathrm{ox}} = 34.5\mathrm{nF/cm}^2$

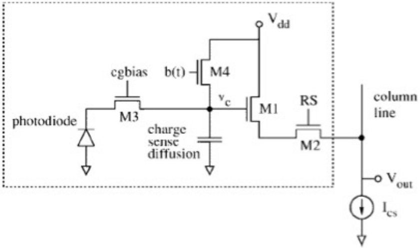
➔ $x_{\mathrm{d}} = 1.5\mu\mathrm{m}$, $V_{\mathrm{surface}} = 3.44\mathrm{V}$

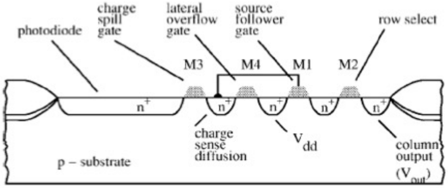
➔ $Q = 1.19\mathrm{x}10^{-15}\mathrm{C}/\mu\mathrm{m}^2$; $WCAP = 2.4\mathrm{x}10^5\mathrm{e}^-$

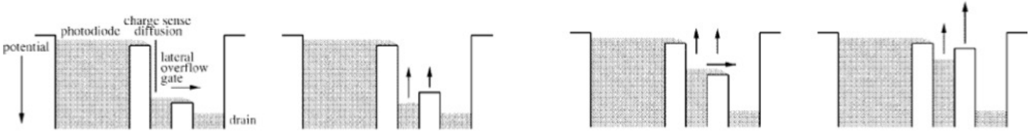
2.4.2 Dynamic Range Extension

- Adjust well capacity
- Change conversion gain
- Change exposure time
- Make conversion gain non linear

2.4.2 Dynamic Range Extension – Adjust Well Capacity

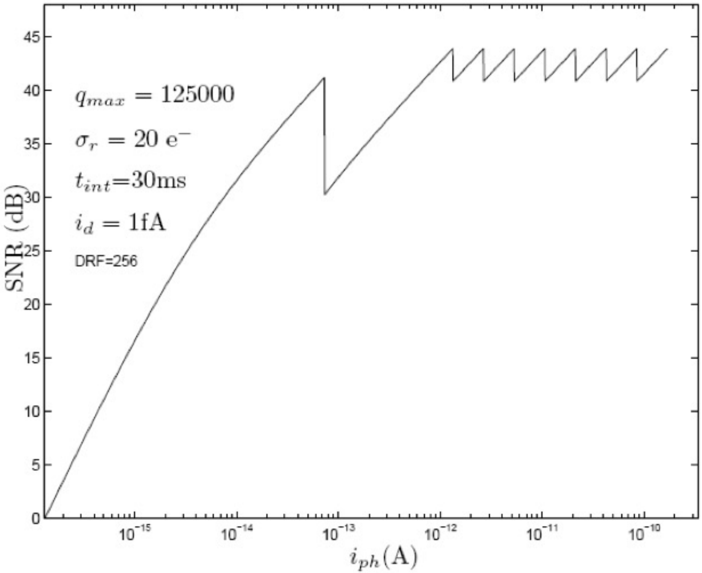






Source: George Yuan

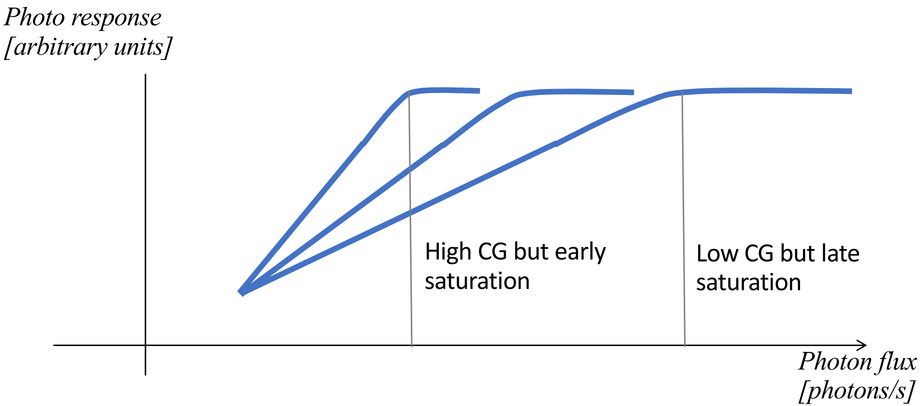
2.4.2 Dynamic Range Extension – Signal-to-noise



Source: George Yuan

2.4.2 Dynamic Range Extension – Change Conversion Gain

- High CG causes saturation in bright scenes
- Low CG does not enable detection of dim scenes
- DR is the same

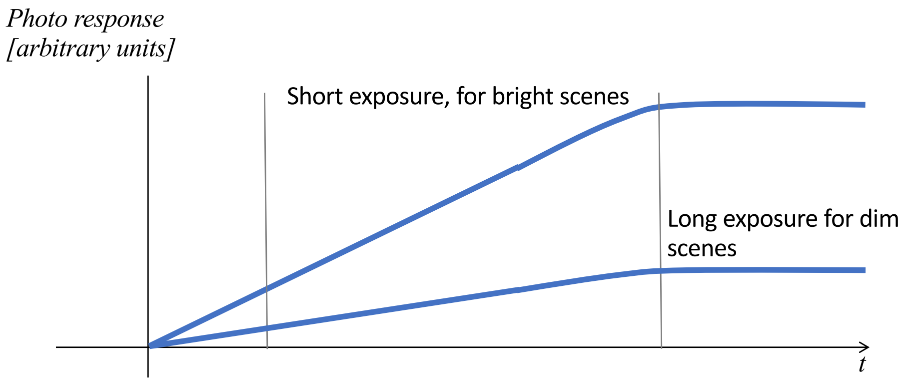


2.4.2 Dynamic Range Extension – Change Conversion Gain

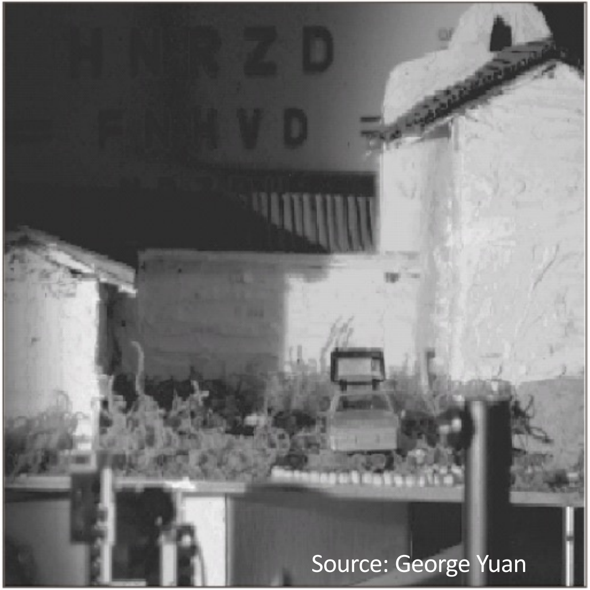
- Take two or more pictures with different CG
- Then, pixel-by-pixel:
  1. detect saturation
  2. Keep pixels with highest CG, unsaturated
  3. Replace saturated pixels with unsaturated pixels with lower CG
  4. Each pixel level must be normalized depending on the specific CG
- *Drawback:* fast-moving objects incorrectly interpolated!

2.4.2 Dynamic Range Extension – Change Exposure Time

- Longer exposure time for dim objects ➔more charges at floating diffusion (but also higher Poisson noise)
- Upper limit pushed up but lower limit unchanged
- <u>*Drawback:*</u> fast moving objects incorrectly interpolated!



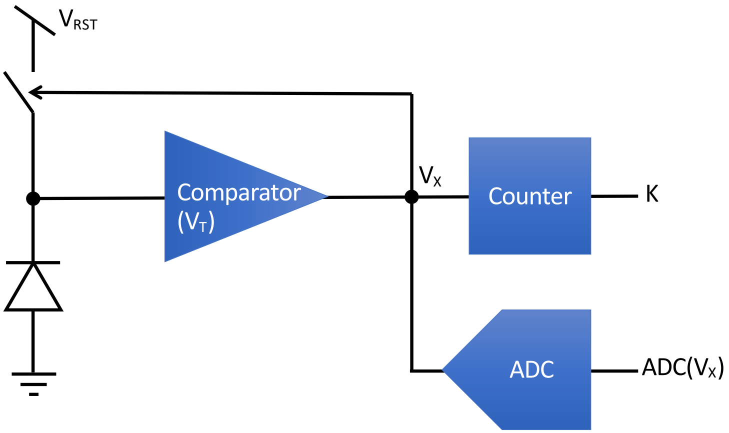
2.4.2 Dynamic Range Extension – MST Example



2.4.2 Dynamic Range Extension – iPhone4 Example

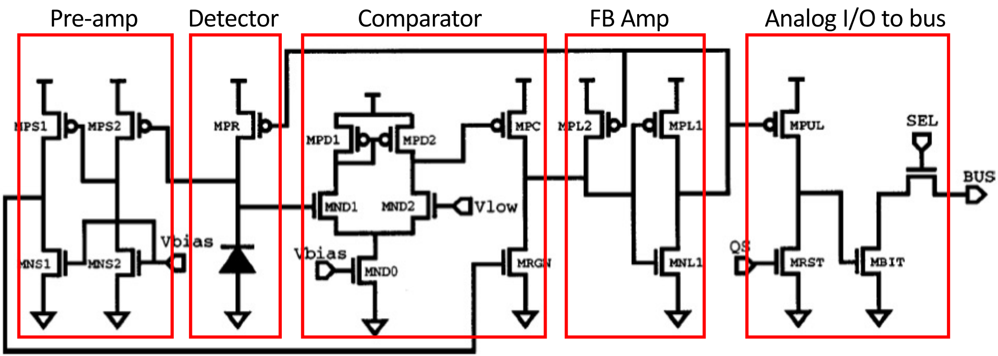
2.4.2 Dynamic Range Extension – Pixel Level Quantization (ΣΔ)

- Drawbacks:
  - Large circuitry (loss of FF)
  - Every reset introduces kTC noise



2.4.2 Dynamic Range Extension – Pixel Level ΣΔ ADC

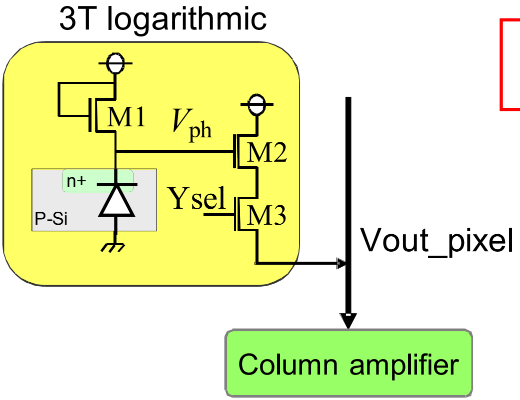
- Implementation at transistor level
- Analog residual output to bus



Source: George Yuan

2.4.2 Dynamic Range Extension – Non-linear (log) Conversion Gain

- M1 operates in sub-threshold regime
- A voltage is generated that is the log of the current of the photodiode



$$V_{ph} \propto \log(I_{ph})$$

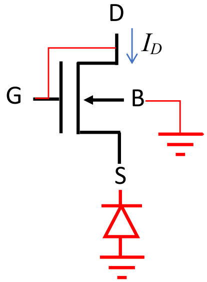
2.4.2 Dynamic Range Extension – Non-linear (log) Conversion Gain

- In weak inversion, I_D is

$$I_D = I_0 \cdot e^{\frac{qV_{GB}}{nkT}} \left(e^{\frac{-qV_{SB}}{kT}} - e^{\frac{-qV_{DB}}{kT}} \right)$$

- With a diode connected NMOS

$$V_{\mathrm{GB}} = V_{\mathrm{dd}};\ V_{\mathrm{SB}} = V_{\mathrm{ph}};\ V_{\mathrm{DB}} = \mathrm{V}_{\mathrm{dd}}.$$



- Finally, by rearranging, we obtain

$$V_{ph} = V_{dd} - \frac{kT}{q} \ln \frac{I_{ph}}{I_0}$$

- As the illumination (and hence I_{ph}) increases linearly, the **output voltage decreases logarithmically**

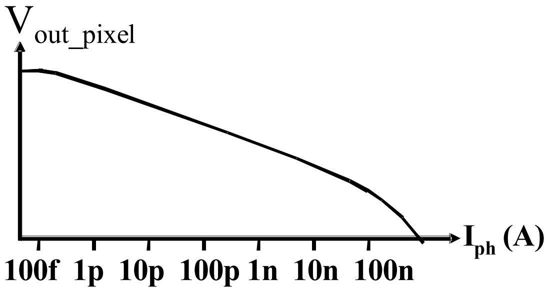
2.4.2 Dynamic Range Extension – Non-linear (log) Conversion Gain

- Drawbacks:

  Non-uniformity of behavior (I_0 changes across the array)

  PVT (process, supply voltage, temperature) variability

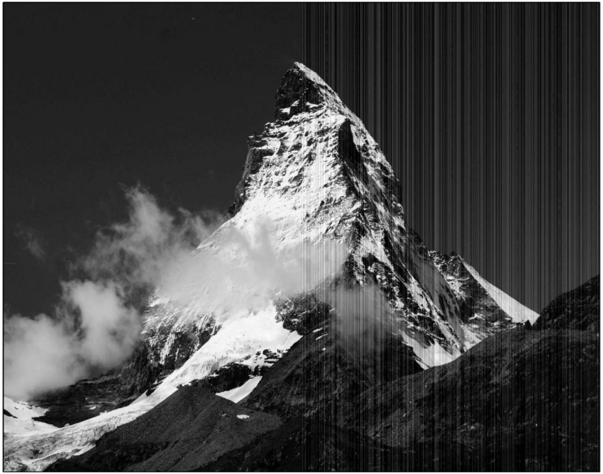
  Low speed at low light levels since the only way of charging / discharging the sensing node is by means of photocurrent



2.4.3 Quiz A

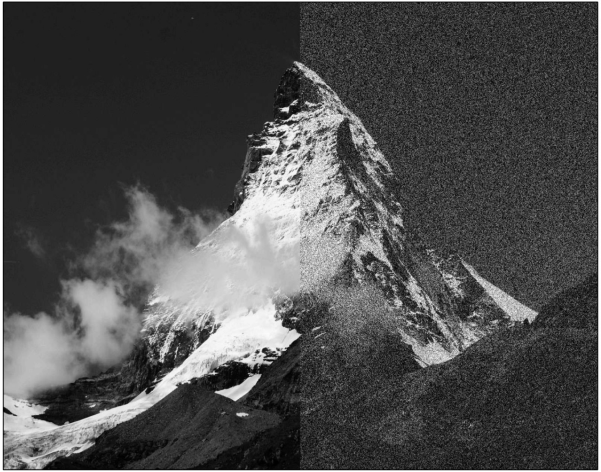
Source: Alice Pelamatti

2.4.3 Quiz B



Source: Alice Pelamatti

2.4.3 Quiz C



Source: Alice Pelamatti

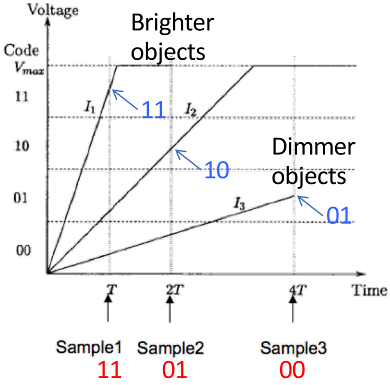
APPENDIX

2.0 QUIZ

- Do you remember what a random process is? Can you define it compactly?
- How do you characterize a random process?
- Can you remember a special property of a random process?

2.4.2 Dynamic Range Extension – Multiple Sampling with Variable T

- A non-destructive sampling is used in this scheme!
- The time at which the last non-saturated sample was taken is recorded relative to the end of the frame.



Illumination	$x_1x_2x_3x_4$	Exponent	Mantissa
I_1	1 1 1 1	2	11
I_2	0 1 0 1	1	10
I_3	0 0 0 1	0	01

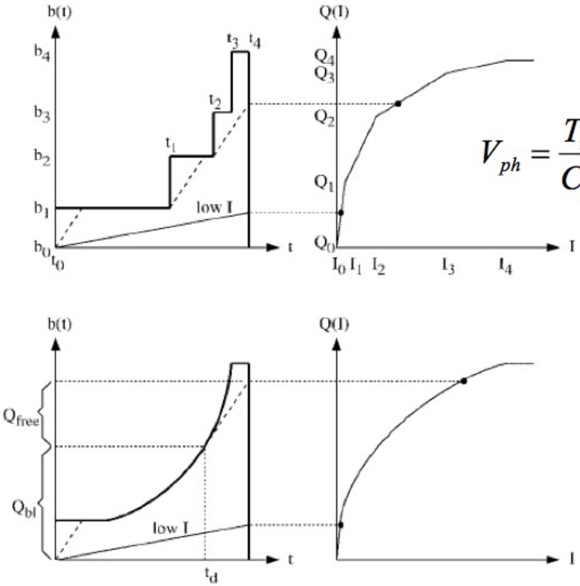
↑ k ↑ m

Source: George Yuan

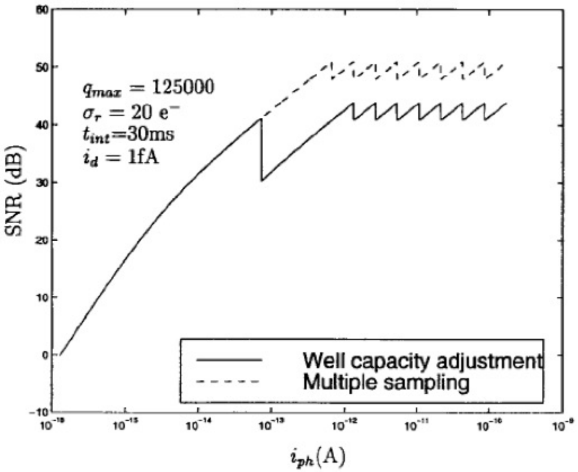
$$V_{ph} = \frac{T_{\text{int}}}{C_{\text{int}}} I_{ph}$$

Exposure T, 2T, 4T, , 2^kT Quantization: m bits Pixel resolution: m+k

2.4.2 Dynamic Range Extension – Multiple Sampling with Variable T



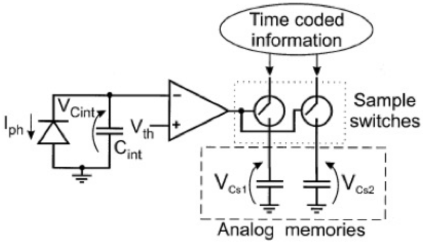
$$V_{ph} = \frac{T_{int}}{C_{int}} I_{ph}$$

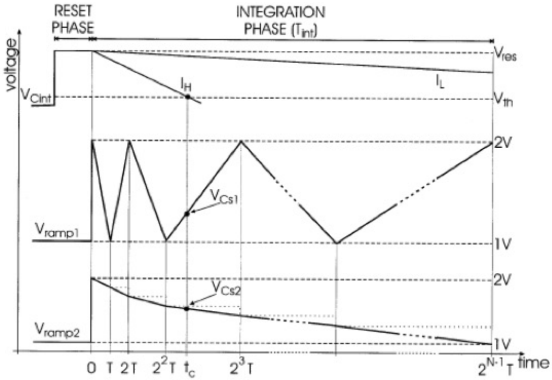


Source: George Yuan

2.4.2 Dynamic Range Extension – Time-to-saturation Pixel

- Similar to a pixel-level ADC, but coding is done with two or more analog counters to have a unique combination!



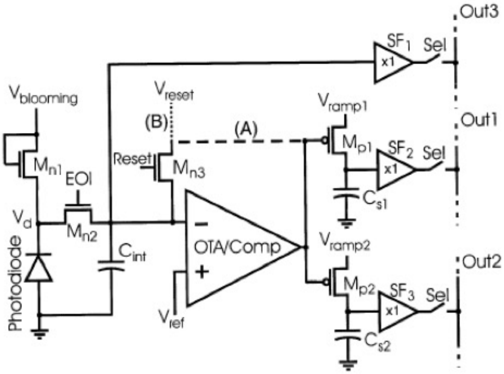


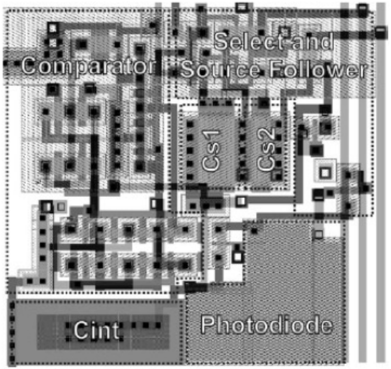
$$V_{ph} = \frac{T_{int}}{C_{int}} I_{ph}$$

Source: George Yuan

2.4.2 Dynamic Range Extension – Time-to-saturation Pixel

- Drawbacks:
  Large circuitry (loss of FF)
  Every reset introduces kTC noise





Source: George Yuan

2.4.2 Dynamic Range Extension – Pixel Level Quantization

- Reset photodiode discharge when threshold is reached. Increment counter (K++) and continue discharge.
- At the end of the frame ($t = T$), the voltage is sampled & held and A/D converted; the final code C is: